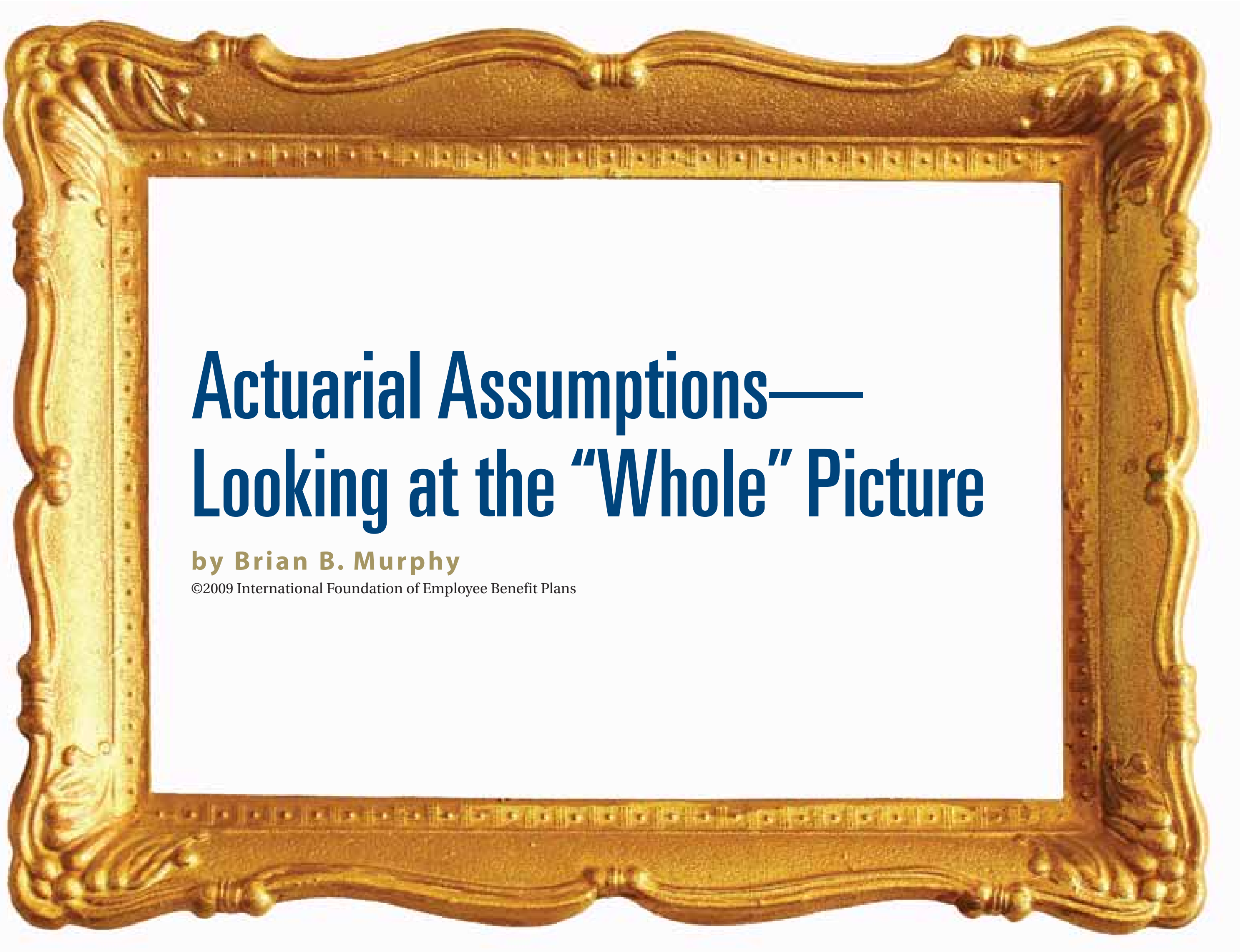
Actuaries go through a complicated set of assumptions to put a value on defined benefit plans. This article explains the actuarial process used in public sector pension plans, starting with a hypothetical employee's situation and then making more general assumptions. The author aims to help trustees understand the many considerations that make up the process.



Actuarial Assumptions— Looking at the "Whole" Picture

by Brian B. Murphy

This article provides pension plan trustees a detailed look into the process public sector actuaries use to perform actuarial valuations of defined benefit (DB) plans. An actuarial valuation is a complicated matter involving many assumptions.

Take Mary, for Example

As an example plan, the author uses a pure DB plan that provides a benefit of 1.5% times three-year final average compensation times years of service. The plan also provides a guaranteed 2% compound cost-of-living adjustment (COLA). Many of the calculations are based upon a sample participant—Mary, who is 40 years old, has ten years of service and is earning $50,000 per year on the valuation date. For the initial examples, it is assumed that Mary will retire at the age of 60 and live to be 85 years old. Mary will receive pay increases of 4.5% per year each year prior to retirement and will receive the plan's annual 2% compound COLA each year following retirement. Pay increases and COLA will occur on her birthday, as will every other life event that happens to Mary. Later in the article, the assumptions will be made more general.

In performing a valuation of a DB pension plan, one of the first things an actuary must develop is the present value of future benefits. The *present value* of an amount of money payable in the future is the amount of money that, if we had it today, would accumulate to the amount that will be payable considering:

- Investment return
- Probability that money will be paid.

The first step, of course, is to project Mary's pension. Initially the author will simplify the calculation by assuming that Mary will retire at the age of 60—in other words, she won't quit, become disabled or die before then. The calculation is shown in Table I.

Mary's annual pension (payable monthly) at the age of 60 will be $49,722—exactly 45% of her three-year average salary (because 1.5% times 30 years is 45%), and about 41% of her pay the year she retires. Notice that her pay more than doubles in the 20 years between the valuation date and the retirement date, partly due to the effects of inflation. Notice the steep rate of increase in the accrued pension at younger ages. Part of the increase is due to earning an extra year of service, and part of it is due to the increase in Mary's final average pay.

In calculating the present value of Mary's pension, the author uses an interest rate assumption of 8% per year. To calculate the present value, considerations must include the amount of the payment to be made each year, the probability that the payment will be made and the interest discount. Since the assumption is that Mary will live to exactly age 85, the probability of payment is 100% until then and 0% thereafter. The results are shown in Table II. In the table, the payment stream is initially discounted back to Mary's retirement age (60), and is then further discounted back to her age on the valuation date (age 40).

Now that the basic present value calculation has been shown in a general but relatively simple example, the author discusses more general assumptions that are similar to those actuaries actually use in calculations. Table III outlines the key assumptions actuaries use in pension plan valuations.

Economic Assumptions

The key economic assumptions are the investment return assumption (called the *interest rate* or sometimes the *discount rate*), and the payroll growth rate. It is important that all of the economic assumptions are based on the same underlying price inflation assumption.

The interest rate assumption typically consists of these components:

- A risk-free rate consisting of a portion such as 3% due to pure price inflation and a small risk-free or low-risk real return rate in the range of 0.5% to 1.5%
- An equity risk premium such as 3.5% representing an assumed reward for taking equity risk.

The pay increase assumption typically consists of three parts:

1. A component such as 3% assumed to be due to pure price inflation
2. A component such as 1% assumed to be due to productivity and living standard improvements
3. A component varying by age or service that reflects the individual's performance.

When Mary's pension was projected, it was assumed that Mary would always get a 4.5% pay increase per year. In the above context, that might have arisen from a 3% inflation assumption, a 1% general productivity assumption and a 0.5% individual performance assumption. Such an assumption would be called a "flat" salary scale. Table IV uses a general pay increase assumption to project the final average earnings of a person who is hired at $35,000 per year and works to the end of a 35-year career.

The Payroll Growth Assumption

When unfunded liabilities are being amortized as a level percentage of payroll (very common in governmental plans), an assumption must be made regarding the rate of payroll growth. If a constant population is assumed (almost always the case), the payroll growth assumption will usually be the sum of the assumed inflation rate and the assumed productivity component of the salary scale.

In these examples, the payroll growth assumption would be 4% (3% plus 1%). In today's world, governments are lucky if their budgets grow with price inflation. Some actuaries reason that if unfunded liabilities are funded as a level percentage of pay and the payroll growth assumption is higher than inflation, underfunding can result. Actuaries with that viewpoint tend to choose a payroll growth assumption that is close to the price inflation component of the salary scale and does not include the productivity component. They would use 3% as the payroll growth assumption in our example. The author uses 4% in his examples, but respects the other point of view.

Demographic Assumptions

Up until this point, highly simplified assumptions have been used in calculating present values related to Mary. Recall that Mary is 40 years old and has ten years of service. The assumption was that she would work until the age of 60 and be eligible for a pension of $49,722 with a present value at the age of 40 of $140,508. Her present value might not actually be that high, because she might not stay in service to retirement. If she doesn't, her present value would probably go down. For example, she could quit, leave her contributions on deposit and draw her vested benefit when she is old enough (60). She might quit and forfeit her right to a vested benefit by withdrawing her contributions. She could die before retirement. Maybe a survivor benefit would be payable, maybe not. She could become disabled and start drawing a disability benefit. Let's refigure the present value of her pension supposing that the only thing she can do prior to retirement is to quit with a vested pension.

Table V considers the probability that she might quit between now and the age of 60, leave her contributions, if any, on deposit, and come back at the age of 60 to draw the vested benefit with COLA commencing after retirement.

The table uses a scale of quit probabilities starting at 3% and declining to 0% in stages. At the age of 40, there is a 3% chance that she will quit. If she does, she will be eligible for a pension of $6,872 at the age of 60. The present value of that benefit is $582. Referring to figures from both Tables V and II, this figure can be calculated as: $582 = $140,508/$49,722 x $6,872 x 1.00 x 0.03. In this calculation, 1.00 is the probability of being in service at the age of 40 and 0.03 is the probability of quitting. Of course, the probability of being in service at the age of 41 is now only 0.97 since there was a 3% chance she would quit at the age of 40.

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Table I

Age	Service	Pay: Current	Pay: 3-Year Avg.	Accrued Pension	% Increase
40	**10**	**$ 50,000**	**$ 45,816**	**$ 6,872**	
41	11	52,250	47,878	7,900	15%
42	12	54,601	50,033	9,006	14
43	13	57,058	52,284	10,195	13
44	14	59,626	54,636	11,474	13
45	15	62,309	57,095	12,846	12
50	20	77,649	71,151	21,345	10
55	25	96,764	88,667	33,250	9
60	**30**	**$120,585**	**$110,494**	**$49,722**	**8**

At the age of 60, Mary's pension is projected to be $49,722 per year. But look at the rate of increase in the accrued pension!

Table II

Age	Pension	8% Discount: Factor	8% Discount: PV Inc.
60	**$49,722**	**0.96225**	**$47,845**
61	50,716	0.89097	45,187
62	51,731	0.82497	42,677
63	52,765	0.76387	40,306
64	53,821	0.70728	38,066
65	54,897	0.65489	35,952
70	60,611	0.44571	27,015
75	66,919	0.30334	20,299
80	73,884	0.20645	15,253
85	—	0.14051	—
Present Value at Retirement (Age 60)			**$654,901**
Discount to Valuation Date			0.21455
Present Value at Valuation Date (Age 40)			**$140,508**

Table III

What: Economic	What: Demographic
• Investment Return	• Retirement Rates
• Payroll Growth Rate	• Promotional/Step Pay Increases
• Population Growth Rate (Usually, a constant population size is assumed.)	• Disability
	• Turnover
	• Mortality
Who Selects	
• It depends . . .	• Mostly Actuary

At this point, sufficient information is available to calculate her total present value taking into account the fact that she might quit between age 40 and retirement age. Table V shows that under these assumptions, the probability of her retiring directly from active service is 0.6504. The calculation is shown in Table VI.

What If Mary Dies or Becomes Disabled?

Each time an additional preretirement event or *decrement* is introduced, the actuary estimates the probability of the event happening at each age or service, estimates the present value associated with the event at that age or service, and then sums the values over all the possible ages or services. In most (but not all) cases, the introduction of another decrement lowers the total present value of benefits yet further.

The Retirement Assumption

Up to this point, it has been assumed Mary would retire at the age of 60 with 100% probability. More typically, actuaries will assume a scale of retirement probabilities, depending on age, service or both in some way. Table VII shows a typical retirement assumption.

When people choose to retire later than first eligibility for full benefits, plan costs usually are reduced because they will draw the benefit for fewer years. Also,

Table IV

	Pay Increase Due to				Pay at Year End		Real Pay
Service	Inflation	Productivity	Merit	Total	Beg. of Year	End of Year	(Price Adj.)
0	3%	1%	3.0%	7.0%	$35,000	$37,450	$36,359
1	3	1	3.0	7.0	37,450	40,072	37,772
2	3	1	3.0	7.0	40,072	42,877	39,239
3	3	1	3.0	7.0	42,877	45,878	40,762
4	3	1	3.0	7.0	45,878	49,089	42,345
5	3	1	2.5	6.5	49,089	52,280	43,784
6	3	1	2.5	6.5	52,280	55,678	45,271
7	3	1	2.5	6.5	55,678	59,297	46,810
8	3	1	2.5	6.5	59,297	63,151	48,400
9	3	1	2.5	6.5	63,151	67,256	50,045
10	3	1	2.0	6.0	67,256	71,291	51,502
15	3	1	1.5	5.5	90,002	94,952	59,171
20	3	1	1.0	5.0	117,629	123,510	66,393
25	3	1	0.0	4.0	150,128	156,133	72,398
30	3	1	0.0	4.0	182,653	189,959	75,981
35	3	1	0.0	4.0	222,224	231,113	79,741

The inflation and productivity assumptions do not depend on service (or age) since they are macroeconomic assumptions.

Table V

		Probability of			Present
				Accrued	Value at
Age	Service	Being There	Quitting	Pension	8%
40	**10**	**1.0000**	**3.0%**	**$ 6,872**	**$ 583**
41	11	0.9700	3.0	7,900	702
42	12	0.9409	3.0	9,006	838
43	13	0.9127	3.0	10,195	994
44	14	0.8853	3.0	11,474	1,172
45	15	0.8587	2.5	12,846	1,145
50	20	0.7566	2.0	21,345	1,971
55	25	0.6839	1.0	33,250	2,039
56	26	0.6771	1.0	36,136	2,369
57	27	0.6703	1.0	39,215	2,748
58	28	0.6636	1.0	42,497	3,185
59	29	0.6570	1.0	45,995	3,685
60	**30**	0.6504	0.0%	**$49,722**	—
Present Value of Deferred Vested Quit Decrement					**$39,847**

people working longer results in a smaller retired life group and less benefit payout in total (even if each person gets more).

When early reduced retirement is added to a plan, however, costs often rise. Early retirees will work fewer years and will draw the benefit for more years. People working fewer years results in a larger retired life group and (usually) more benefit payout in total (even if each person gets less!).

Death After Retirement

The final assumption is the death-after-retirement assumption. This decre-

Continued on next page

Table VI

1.	Present value of benefit if Mary retires directly from active service	$140,508
2.	Probability that Mary retires directly from active service	0.6504
3.	Present value of retirement: 1×2	91,388
4.	Present value of vesting benefit	39,847
5.	Total present value (3+4)	$131,235
6.	Percent of original calculation: 5/1	93%

Table VII

Age	Probability	Type of Benefit
55	5%	Reduced
56	5	Reduced
57	5	Reduced
58	5	Reduced
59	5	Reduced
60	25	Full
61	10	Full
62	20	Full
63	20	Full
64	20	Full
65	35	Full
66	35	Full
67	15	Full
68	15	Full
69	15	Full
70	100	Full

ment eventually affects everyone. Of course, many people elect optional forms of benefits protecting a spouse or other survivor, so the effect on the plan can be delayed into the far future. Actuaries almost always assume different mortality rates for men and women and often distinguish disabled retirees from early or normal retirees. Table VIII shows a typical modern mortality table that could be applicable to normal retirees.

Application to Mary

It has been assumed Mary would die at exactly age 85 (on her birthday!). Table VIII shows that her life expectancy at the age of 60 is only to the age of 84.38. While she might die before the age of 84.38, there is a chance she will live to be 120.5 years old. If all these factors are taken into account properly, the present value of her pension will be lower than if an annuity was calculated assuming she would exactly live to her life expectancy.

Rather than creating still more numbers, let's assume that when everything is taken into account, Mary's present value based on 8% at retirement is still $654,901 and her present value is $140,508. At this point, the actuary must introduce the *actuarial cost method*, which splits the value today—$140,508—into a past and future piece and defines the *normal cost*, which is the portion of the future piece that slips into the past with each passing year of Mary's service. Table IX gives a pictorial description.

Actuarial Cost Methods

The actuarial cost method determines the allocation of cost between past and future. In other words, the *actuarial cost method* is the calculation process that splits the $140,508 into the sum of $65,178 and $75,130. The *entry age normal cost* (EANC) method is the overwhelming choice for funding public employee retirement systems, and the remainder of the article focuses on that method.

The EANC method seeks to answer the following question: What is the level percentage of payroll that, if contributed regularly from time of plan entry until retirement, will accumulate exactly to the present value of benefits at retirement?

Table X gives an example of the development of the EANC.

Table X was developed assuming that the normal cost will grow at 4% per year. In reality, Mary's normal cost will grow at the same rate as her pay (4.5% per year), and should be calculated that way. Four percent has been used in the illustration in order to indicate how the normal cost behaves in a more general case. In a large plan, with a flow of people into and out of the plan, both the payroll and the normal cost will grow at a rate that is consistent with the macroeconomic assumptions related to inflation and productivity, or 4% in these examples.

The reader can work with the chart a little to gain an understanding of the math involved. In particular:

- $3,995 = $3,841 x 1.04. The normal cost increases 4% per year, exactly in line with the assumed rate of growth of the total payroll. This levels the normal cost across generations.
- $12,950 = $8,144 x 1.08 + $4,155. The accrued liability is the accumulated value of past normal costs. It is the asset value that the plan would have if all of the current assumptions had been exactly met in the past.
- The contribution pattern results in exactly $654,901 at the end of 30 years.

Under this method, the actuarial accrued liability with ten years of service is $65,178 and her normal cost is $5,468 or 10.94% of her pay. The author now supposes that the plan has 1,000 people similar to Mary, and that it currently has $48,530,000 in assets. The unfunded liability and funded ratio would be as shown in Table XI.

An unfunded liability arose, either because there were actuarial losses in the

Table VIII

	Males			Females			
Age	Rate	Life Exp.	Age at Death	Rate	Life Exp.	Age at Death	Diff (F-M)
60	0.006747	21.74	81.74	0.005055	24.38	84.38	2.64
61	0.007676	20.88	81.88	0.005814	23.50	84.50	2.62
62	0.008757	20.04	82.04	0.006657	22.63	84.63	2.59
63	0.010012	19.21	82.21	0.007648	21.78	84.78	2.57
64	0.011280	18.40	82.40	0.008619	20.95	84.95	2.55
65	0.012737	17.61	82.61	0.009706	20.12	85.12	2.51
70	0.022206	13.88	83.88	0.016742	16.23	86.23	2.35
75	0.037834	10.57	85.57	0.028106	12.74	87.74	2.17
80	0.064368	7.75	87.75	0.045879	9.68	89.68	1.93
85	0.110757	5.49	90.49	0.077446	7.09	92.09	1.60
90	0.183408	3.86	93.86	0.131682	5.15	95.15	1.29
95	0.267491	2.84	97.84	0.194509	3.97	98.97	1.13
100	0.344556	2.25	102.25	0.237467	3.29	103.29	1.04
105	0.397886	2.00	107.00	0.293116	2.62	107.62	.62
110	0.400000	1.99	111.99	0.364617	2.13	112.13	.14
115	0.400000	1.88	116.88	0.400000	1.88	116.88	0.00
120	1.000000	.50	120.50	1.000000	.50	120.50	0.00

past, benefits were increased in the past, or the plan sponsor did not make the required contributions. It is now necessary to figure out how to fund the unfunded liability. The plan sponsor, in conjunction with the Retirement Board and actuary, has decided upon 30-year level percentage of pay funding of the unfunded liability. Table XII shows how this works.

Some key items to notice from the chart are:

- The dollar payment against the unfunded liability grows at 4% each year. For example, $983 = $945 x 1.04. If the payroll actually grows at 4% per year, the payment against the unfunded liability will be the same percentage of payroll in every year of the schedule that it was in the first year (1.89%).
- The unfunded liability grows in dollar amount for the first ten years or so and then begins to decrease. This is sometimes referred to as *negative amortization*. It is an expected result of level percent of payroll funding.
- The unfunded liability decreases as a percent of payroll every year.

Enough information has been given to show the results of the entire valuation. See Table XIII.

Technical Issues

There are two technical issues that have been sidestepped up until this point. One relates to the conversion of the normal cost to a percentage of payroll. The other relates to funding the unfunded liabilities when there is a time lag between the valuation date and the contributions that will be received based upon the results. These will be treated one at a time.

Normal Cost Conversion

A common, six-step calculation method for developing the EANC is as follows:

1. Calculate the present value of all future benefits (PVFB) at entry age.
2. Calculate the present value of future salary (PVFS) at entry age.
3. Divide the two figures to get the EANC percentage (EANC% = PVFB/PVFS).
4. Calculate the pay expected to be received by the person in the year following the valuation date.
5. Calculate EANC dollars as EANC percentage times the result of step 4.
6. Sum the results of step 5 over each person in the group. The total of this is the total EANC dollars for the entire group, referred to as TEANC$. Report this result out of the valuation program.

In corporate plans, the calculation tends to stop at step 6, since there is rarely interest in level percent of payroll financing. In public plans, the author's preferred method for converting the results of step 6 to a percentage of payroll is shown below.

7. Sum the results of step 4 above for each person in the group. Call this figure TEXPPAY.
8. Calculate the total EANC percent as EANC%=TEANC$/TEXPPAY.

The author believes that the method illustrated in steps 7 and 8 is most consistent with level percent of payroll financing and that most other methods in use will tend to understate the normal cost as a percent of pay.

Unfunded Liabilities With Time Lag

Unlike with normal cost, the author believes it is correct to calculate the percent of pay required to fund unfunded liabilities by projecting payroll at the full

Continued on next page

Table IX

Present Value of Benefits

At Retirement Date	$654,901
At Valuation Date	$140,508

Allocated to Past and Future Service

$65,178	$75,330
Actuarial Accrued Liability	Present Value of Future Normal Costs

Actuarial Accrued Liabilities
− Accrued Assets

Unfunded Actuarial Accrued Liabilities

Table X

Entry Age Normal Method

4% Pay Growth

Interest Rate 8%

Year	Normal Cost	Accrued Liability
1	$3,841	$3,841
2	3,995	8,144
3	4,155	12,950
4	4,321	18,308
5	4,494	24,266
10	5,468	65,178
20	8,093	237,195
30	11,980	654,901

Table XI

($ Thousands)

Plan Totals	Assumed Interest Rate 8.00%
1. Actuarial Accrued Liability	$65,178
2. Valuation Assets	48,530
3. Unfunded Liability (1)-(2)	$16,648

The funded ratio is usually expressed based upon the actuarial value of assets as shown. An alternate expression based upon the market value of assets can also be helpful.

payroll growth rate (4% in the examples). But there is another issue. Typically in governmental plans, at least, the valuation is done as of a certain date, for example, December 31, 2008. But the results of that valuation will not affect the contribution rate until a year or more later. So if the valuation date is December 31, 2008, the contribution rate determined in the valuation might not take effect until January 1, 2010. When this situation arises, many actuaries will choose to project the unfunded liability one year forward, based upon contributions scheduled to be received in 2009, and to project the payroll two years forward before calculating the percentage of pay to fund the unfunded liabilities. Other actuaries will choose to ignore the time lag and to make calculations as though the rates were going to become effective the day following the valuation date. The author believes that in most cases, the former approach is a better approach.

A Few Other Issues

Actuaries also must make assumptions that take into account the following:

- Rates of option election
- Rates of service purchases
- Timing of pay increases within the year
- Timing of retirements and other decrements within the year.

Option elections and service purchases—particularly the latter—often are not cost neutral. Even today, actuaries sometimes do not account for these activities in their work. The effects of service purchases can be significant and sometimes difficult to isolate.

Conclusion

An actuarial valuation is a complicated, technical process that can be completely understood only by people who have spent years studying the underlying mathematics, and who have considerable hands-on experience in the industry. With this brief article, the author hopes to have made some parts of the valuation process more accessible to trustees than the literature up to this point has done. The author believes that trustees who expend the necessary time and effort to learn the material in this article will be able to participate more effectively in actuarially related trustee decisions than others. The author believes that such informed participation by trustees will enhance the decision-making process and help to provide secure retirement benefits for plan participants. **B&C**

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Table XII

($ Thousands)

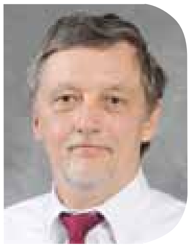
Year	UAL Beg of Year	Payment	Interest	UAL End of Year	Payroll	Percent of Payroll: UAL Contrib	Percent of Payroll: UAL
1	$16,648	$945	$1,295	$16,997	$50,000	1.89%	33%
2	16,997	983	1,321	17,335	52,000	1.89	33
3	17,335	1,022	1,346	17,659	54,080	1.89	32
4	17,659	1,063	1,371	17,966	56,243	1.89	31
5	17,966	1,106	1,394	18,254	58,493	1.89	31
10	19,137	1,345	1,478	19,269	71,166	1.89	27
15	19,283	1,637	1,478	19,125	86,584	1.89	22
20	17,585	1,992	1,328	16,921	105,342	1.89	17
25	12,760	2,423	925	11,262	128,165	1.89	10

With 30-year level percent of payroll amortization, the absolute value of the unfunded liability will increase for several years before it finally begins to decrease. It always decreases, though, as a percent of payroll if all assumptions are met.

Table XIII

($ Thousands)

Contribution for	Assumed Interest Rate 8%: $ Amount	Assumed Interest Rate 8%: % of Payroll
Normal Cost	$5,468	10.94%
Unfunded Liability	945	1.89
Total	6,413	12.83



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